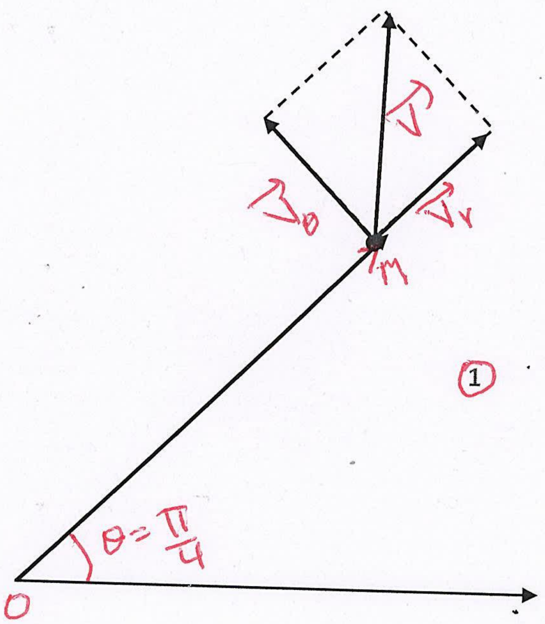
A typical correction of the Physics 1 Exam

1. the position vector $\overrightarrow{OM}$:

$$\overrightarrow{OM} = r\vec{e}_r = 2\sin(\theta)\vec{e}_r \quad 1$$

The representation of $\overrightarrow{OM}$ in the (polar) coordinate system at **t = 1s**. Scale : **1cm → 0.2m.**

At t=1s $\begin{cases} \theta(1) = \pi/4 \\ r(1) = 2\sin(\pi/4) = 1.41m \rightarrow r(1) \approx 7.07cm \end{cases}$ 0.5



2. the radial $\vec{V_r}$ and transversal $\vec{V_\theta}$components of the velocity :

$$\vec{V} = \frac{d\overrightarrow{OM}}{dt} = \dot{r}\vec{e}_r + r\dot{\vec{e}}_r = 2\,\dot{\theta}\cos(\theta)\vec{e}_r + 2\,sin(\theta)\,\dot{\theta}\vec{e}_\theta$$

Where $\dot{\theta} = \frac{d\theta}{dt} = \pi\, t/2$

$$\vec{V} = \pi t\cos(\theta)\vec{e}_r + \pi t\sin(\theta)\,\vec{e}_\theta$$

$$\vec{V_r} = \pi t\cos(\theta)\vec{e}_r \quad 0.5$$

$$\vec{V_\theta} = \pi t\sin(\theta)\,\vec{e}_\theta \quad 0.5$$

The representation of $\vec{V}$ in the (polar) coordinate system at **t =1s**. Scale: **1cm→1m/s.**

At t=1s $\begin{cases} |\vec{V_r}(1)| = \pi(1)cos(\pi/4) = 2{,}22\,m/s \rightarrow 2.22cm \\ |\vec{V_\theta}(1)| = \pi(1)sin(\pi/4) = 2{,}22\,m/s \rightarrow 2.22cm \end{cases}$ 0.5

3.

a. The expression of $|\vec{V}|$ at time **t**.

$$|\vec{V}| = \sqrt{(\pi tcos(\theta))^2 + (\pi tsin(\theta))^2} = \pi t\sqrt{(cos(\theta))^2 + (\pi sin(\theta))^2} = \pi t\ m/s \quad 1$$

b. $|\vec{a_t}|$:the magnitude of the tangential component of the acceleration vector at t=1(s).

$$|\vec{a_t}| = \frac{d|\vec{V}|}{dt} = \pi\ m/s^2 \quad 1$$

The radius of curvature at this instant:

$$R_c = \frac{|\vec{V}|^2}{|\vec{a_n}|}$$

Where :

$$|\vec{a_n}| = \sqrt{|\vec{a}|^2 - |\vec{a_t}|^2}$$

Thane we must compute $|\vec{a}|$:

$$\vec{a} = \frac{d\vec{V}}{dt} = \pi cos(\theta)\vec{e}_r - \pi t\dot{\theta} sin(\theta)\, \vec{e}_r + \pi t cos(\theta)\dot{\theta}\vec{e}_\theta + \pi sin(\theta)\, \vec{e}_\theta + \pi t\dot{\theta} cos(\theta)\vec{e}_\theta - \pi t sin(\theta)\dot{\theta}\, \vec{e}_r$$

$$\vec{a} = (\pi cos(\theta) - \pi^2 t^2 sin(\theta))\vec{e}_r + (\pi sin(\theta) + \pi^2 t^2 cos(\theta))\, \vec{e}_\theta$$

$$\vec{a} = \sqrt{(\pi cos(\theta) - \pi^2 t^2 sin(\theta))^2 + (\pi sin(\theta) + \pi^2 t^2 cos(\theta))^2}$$

$$\vec{a} = \sqrt{\pi^2 cos^2(\theta) + \pi^4 t^4 sin^2(\theta) + \pi^2 sin^2(\theta) + \pi^4 t^4 cos^2(\theta)}$$

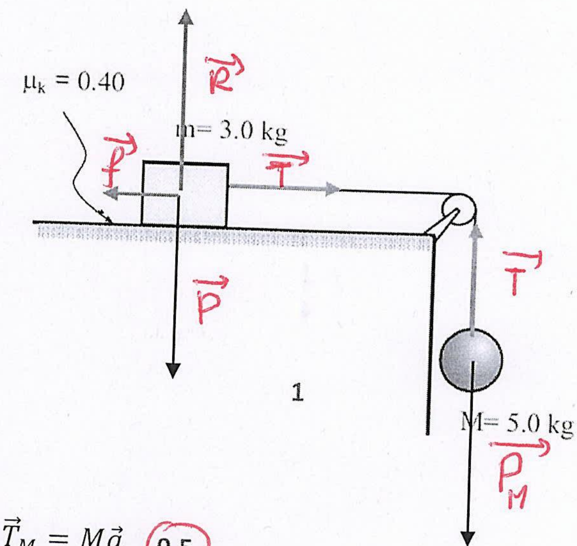
$$\vec{a} = \sqrt{\pi^2 + \pi^4 t^4} \quad 1$$

$$|\vec{a_n}| = \sqrt{\pi^2 + \pi^4 t^4 - \pi^2} = \pi^2 t^2\ m/s^2$$

$$R_c = \frac{|\vec{V}|^2}{|\vec{a_n}|} = \frac{(\pi t)^2}{\pi^2 t^2} = 1\ m \quad 1$$

Exercise 2:

a) . the acceleration of mass m:



By applying Newton's, second law on the mass m:

$$\sum \vec{F}_{net} = m\vec{a} \Rightarrow \vec{p} + \vec{R} + \vec{T}_m + \vec{f} = m\vec{a} \quad 0.5$$

By projection of this equation on the axis:

$$\begin{cases} OX: T_m - f = ma & 0.5 \\ Oy: R - mg = 0 \Leftrightarrow R = mg & 0.5 \end{cases}$$

We have:

$$\mu_k = \frac{f}{R} \Rightarrow f = \mu_k R = \mu_k mg \quad 0.5$$

$$T_m - \mu_k mg = ma$$

By applying Newton's, second law on the mass M:

$$\sum \vec{F}_{net} = M\vec{a} \Rightarrow \vec{P} + \vec{T}_M = M\vec{a} \quad 0.5$$

By projection of this equation on the axis:

$$O'X': Mg - T_M = Ma \quad 0.5$$

The cord and pulley have negligible masses:

$$T_m = T_M = T \quad 0.5$$

by substituting this relation into:

$$\begin{cases} T - \mu_k mg = ma \\ Mg - T = Ma \end{cases} \Rightarrow a = \frac{Mg - \mu_k mg}{M + m} = \frac{5 \times 10 - 0.4 \times 3 \times 10}{5 + 3} = 4.75\ m/s^2 \quad 0.5$$

(b) the tension in the cord: from the equation ():

$$T = Mg - Ma = 5 \times 10 - 5 \times 4.75 = 26.25N \quad 0.5$$

Exercise 3:

The work done by gravity:

$$W_{AB}(\vec{P}) = mgh = -mgABsin(30) = -15 \times 10 \times 5 \times 0.5 = -375J$$ 0.5

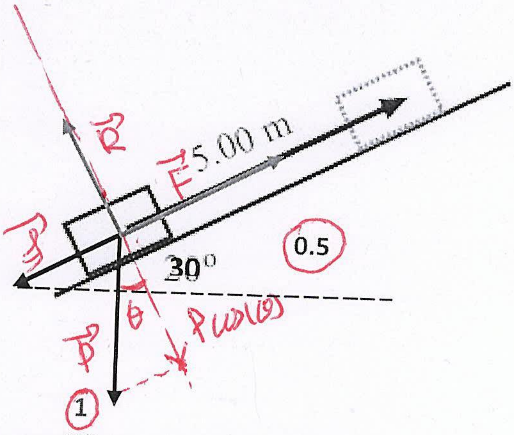
Energy lost due to friction:

$$W_{AB}(\vec{f}) = \vec{f} \cdot \overrightarrow{AB} = fABcos(180) = -fAB$$

While:

$$\mu_k = \frac{f}{R} \Rightarrow f = \mu_k R, and\ R = mgcos(30) \Rightarrow f$$
$$= \mu_k mgcos(30) = 0.4 \times 15 \times 10 \times 0.86$$
$$= 52N$$ 0.5

$$W_{AB}(\vec{f}) = -fAB = 52 \times 5 = -260J$$ 0.5



work done by the 200 N force:

$$W_{AB}(\vec{F}) = \vec{F} \cdot \overrightarrow{AB} = FABcos(0) = FAB = 200 \times 5$$
$$= 1000J$$ 0.5

the change in kinetic energy of the crate:

$$\Delta E_C = \sum W(\vec{F}_{net}) = \sum W(\vec{F}^{C} + \vec{F}^{NC}) = W_{AB}(\vec{P}) + W_{AB}(\vec{R}) + W_{AB}(\vec{f}) + W_{AB}(\vec{F})$$

Where:

$$W_{AB}(\vec{R}) = 0J$$ 0.5

$$\Delta E_C = -375 - 260 + 1000 = 365J$$ 0.5

the speed of the crate after being pulled 5.00 m

$$\Delta E_C = \frac{1}{2}mV_B^2 - \frac{1}{2}mV_A^2 \Rightarrow V_B = \sqrt{\frac{2\Delta E_C}{m} + V_A^2} = \sqrt{\frac{2 \times 365}{15} + (2.5)^2} = 7.41m/s$$ 1